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MATHEMATICS.—*On Fermat's Last Theorem.*[1] VAL. MAR. SPUNAR, Chicago, Ill. (Communicated by PAUL R. HEYL.[2])

It is well known that the equation

$$x^n + y^n = z^n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1)$$

n being any positive integer, cannot be solved in integers if n be a multiple of 4. If, therefore the equation (1) has integral solutions for any n this exponent must contain an odd prime, say λ, where $\lambda \geq 3$. If then the equation (1) has an integral solution (x, y, z) the equation

$$x^\lambda + y^\lambda = z^\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (2)$$

will have the solution (x^m, y^m, z^m) where $m = n/\lambda$.

If the equation (2) has a solution, it evidently has one in which x, y, z are relatively prime. We shall suppose that one of these solutions is that we are considering: Then, since

[1] Received May 10, 1928.

[2] *Note.*—In the Mathematical Gazette for October, 1923, the undersigned showed that in Fermat's equation (2), for a given value of z, there is a critical value of the exponent λ above which no integral solution is possible. But since this critical value of λ is an increasing function of z, this result leads to no general proof of Fermat's Theorem. In the following article Mr. Spunar shows that no integral solution of Fermat's equation is possible unless z is greater than a certain function of λ, or conversely unless λ is less than a critical value, which is also an increasing function of z. Mr. Spunar's critical limit is far more stringent than that of the writer, and is thus a contribution to the literature of this subject along an almost unworked line.—*P. R. Heyl.*

$$\frac{z^\lambda - y^\lambda}{z-y} = z^{\lambda-1} + yz^{\lambda-2} + \ldots\ldots + y^{\lambda-2}z + y^{\lambda-1}$$

$$= y^{\lambda-1}\left[1 + \frac{z}{y} + \frac{z^2}{y^2} + \ldots\ldots + \frac{z^{\lambda-1}}{y^{\lambda-1}}\right]$$

$$= y^{\lambda-1}\left[1 + 1 + \frac{z-y}{y} + 1 + \frac{z^2-y^2}{y^2} + \ldots\ldots + 1 + \frac{z^{\lambda-1} - y^{\lambda-1}}{y^{\lambda-1}}\right]$$

$$= y^{\lambda-1}\left[\lambda + \frac{z-y}{y} + \frac{z^2-y^2}{y^2} + \ldots\ldots + \frac{z^{\lambda-1} - y^{\lambda-1}}{y^{\lambda-1}}\right]$$

$$= \lambda y^{\lambda-1} + (z-y)y^{\lambda-2} + (z^2-y^2)y^{\lambda-3} + \ldots\ldots + (z^{\lambda-1} - y^{\lambda-1})$$

$$= \lambda y^{\lambda-1} + (z-y)\,U$$

$$= I$$

where U is an integral function of y and z, I and $z - y$ can have no common factor except λ. If then

$$x^\lambda = z^\lambda - y^\lambda = (z-y)\,I$$

any factor of $z - y$ other than λ must divide x^λ, and there must be λ such factors if there is one; whence it follows that $z - y$ is a perfect λth power, unless $z - y \equiv 0 \pmod{\lambda}$.

If then (still excluding the λ factor), $z - y = \alpha^\lambda$ it follows that $x = \alpha\,\xi$.

Since the same reasoning applies whether x, y, or z be taken, we may set

$$x = \alpha\xi \quad \alpha^\lambda = z - y \quad \xi^\lambda = \frac{z^\lambda - y^\lambda}{z-y} \ldots\ldots\ldots\ldots (3)$$

$$y = \beta\eta \quad \beta^\lambda = z - x \quad \eta^\lambda = \frac{z^\lambda - x^\lambda}{z-y} \ldots\ldots\ldots\ldots (4)$$

$$z = \gamma\zeta \quad \gamma^\lambda = x + y \quad \zeta^\lambda = \frac{x^\lambda + y^\lambda}{x+y} \ldots\ldots\ldots\ldots (5)$$

where α, β, γ, ξ, η, ζ, and λ are all relative primes.

It follows, then, that

$$2z = \gamma^\lambda + \beta^\lambda + \alpha^\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (6)$$

$$2y = \gamma^\lambda + \beta^\lambda - \alpha^\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (7)$$

$$2x = \gamma^\lambda - \beta^\lambda + \alpha^\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (8)$$

$$2d = \gamma^\lambda - \beta^\lambda - \alpha^\lambda \quad \ldots\ldots\ldots\ldots\ldots\ldots (9)$$

where

$$d = x + y - z = \alpha(\xi - \alpha^{\lambda-1}) = \beta(\eta - \beta^{\lambda-1}) = \gamma(\gamma^{\lambda-1} - z). \quad (9\ bis)$$

and

$$d = \alpha\beta\gamma\Delta \quad \ldots\ldots\ldots\ldots\ldots\ldots (10)$$

Since

$$\xi^\lambda = \frac{(z - y + y)^\lambda - y^\lambda}{z - y} = (z - y)^{\lambda-1} + \lambda P$$

where P is an integer, and by Fermat's minor theorem

$$(z - y)^{\lambda-1} \equiv 1 \pmod{\lambda}$$

it follows that

$$\xi^\lambda \equiv 1 \pmod{\lambda}$$

Moreover, $(h\lambda + 1)^{\lambda-1} \equiv 1 \pmod{\lambda^2}$ by Fürtwangler's theorem. Multiply both members by $h\lambda + 1$ and apply $(h\lambda + 1)^\lambda \equiv 1 \pmod{\lambda^2}$ and we find that $h \equiv 0 \pmod{\lambda}$ and $\xi = h_1\lambda^2 + 1$.

Similar results can be found for η and ζ. Hence:

$$\xi = 2h_1\lambda^2 + 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

$$\eta = 2h_2\lambda^2 + 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

$$\zeta = 2h_3\lambda^2 + 1 \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

Substituting these values of ξ, η, and ζ in equation (2) and developing by the binomial theorem, we obtain:

$$\gamma^\lambda - \alpha^\lambda - \beta^\lambda \equiv 0 \pmod{\lambda^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

and by (9), (10), and (14)

$$d = \alpha\beta\gamma M \lambda^{3+e} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (15)$$

where α, β, γ, M and λ are relatively prime.

Employing equation (2) in the form

$$(z-y)\left(\frac{z^\lambda - y^\lambda}{z-y}\right) + (z-x)\left(\frac{z^\lambda - x^\lambda}{z-x}\right) = (x+y)\left(\frac{x^\lambda + y^\lambda}{x+y}\right) \ldots (2\ bis)$$

and applying (3), (4), (5), (11), (12), (13), we obtain

$$(z-y)(2h_1\lambda^2+1)^\lambda + (z-x)(2h_2\lambda^2+1)^\lambda = (x+y)(2h_3\lambda^2+1)^\lambda$$

Expanding and reducing, we have

$$(z-y) + (z-x) \equiv (x+y) \pmod{\lambda^3}$$

or

$$\alpha^\lambda + \beta^\lambda \equiv \gamma^\lambda \pmod{\lambda^3}$$

Therefore

$$x + y \equiv z \pmod{\lambda^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (I)$$

Employing (3) in the form

$$x^\lambda = (z-y)\xi^\lambda = (z-y)(2h_1\lambda^2+1)^\lambda$$

$$\equiv (z-y) \pmod{\lambda^3} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (II)$$

and subtracting (II) from (I) we get $x^\lambda \equiv x \pmod{\lambda^3}$. Doing similarly for y and z, we obtain

$$x^{\lambda-1} \equiv y^{\lambda-1} \equiv z^{\lambda-1} \equiv 1 \pmod{\lambda^3} \quad \ldots\ldots\ldots\ldots (16)$$

$$\alpha^{\lambda-1} \equiv \beta^{\lambda-1} \equiv \gamma^{\lambda-1} \equiv 1 \pmod{\lambda^2} \quad \ldots\ldots\ldots\ldots (17)$$

Again, substituting the values of ξ, η and ζ from (11), (12), and (13) into (9 *bis*), viz.,

$$x + y = z + d \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

we obtain, after an easy reduction

$$\gamma = \alpha + \beta \pm 2\lambda^2\left(h_1\alpha + h_2\beta - h_3\gamma - \frac{\alpha\beta\gamma\Delta\lambda^{1+e}}{2}\right) \ldots (19)$$

whence $\alpha + \beta = \gamma \pm \delta$, where δ is any positive integer greater than zero; for if $\delta = 0$, then, by (19), $\gamma = \alpha + \beta$. Equations (6), (7), and (8) give

$$2\alpha\xi = (\alpha+\beta)^\lambda - \beta^\lambda + \alpha^\lambda = 2\alpha^\lambda + \lambda\alpha^{\lambda-1}\beta + \ldots + \lambda\alpha\beta^{\lambda-1} \quad \ldots(20)$$

$$2\beta\eta = (\alpha+\beta)^\lambda + \beta^\lambda - \alpha^\lambda = 2\beta^\lambda + \lambda\alpha\beta^{\lambda-1} + \ldots + \lambda\alpha^{\lambda-1}\beta \quad \ldots(21)$$

$$2\gamma\zeta = (\alpha+\beta)^\lambda + \beta^\lambda + \alpha^\lambda = 2(\alpha+\beta)^\lambda - \lambda\beta(\alpha+\beta)^{\lambda-1} + \ldots + \lambda(\alpha+\beta)\beta^{\lambda-1} \quad \ldots(22)$$

whence

$$\xi = A + \frac{\lambda\beta^{\lambda-1}}{2}, \ \alpha \text{ even} \quad \ldots(20\ bis)$$

$$\eta = B + \frac{\lambda\alpha^{\lambda-1}}{2}, \ \beta \text{ even} \quad \ldots(21\ bis)$$

$$\zeta = C + \frac{\lambda\beta^{\lambda-1}}{2}, \ \gamma = \alpha + \beta \text{ even} \quad \ldots(22\ bis)$$

where A, B, and C are any integers whatsoever. This is, however, contrary to the hypothesis.

If we suppose

$$\alpha + \beta = \gamma + \delta \quad \ldots(23)$$

then, obviously, we have

$$\delta < \alpha < \beta < \gamma \quad \ldots(24)$$

If, however, we assume

$$\alpha + \beta + \delta = \gamma \quad \ldots(25)$$

then it is easy to show that (24) holds good here also. For this purpose we add the following two series of inequalities where $x < y < z$:

$$1 > \frac{x}{z} > \left(\frac{x}{z}\right)^2 > \ldots > \left(\frac{x}{z}\right)^{\lambda-1} > \left(\frac{x}{z}\right)^\lambda > \left(\frac{x}{z}\right)^{\lambda+1} > \ldots > 0 \quad (26)$$

$$1 > \frac{y}{z} > \left(\frac{y}{z}\right)^2 > \ldots > \left(\frac{y}{z}\right)^{\lambda-1} > \left(\frac{y}{z}\right)^\lambda > \left(\frac{y}{z}\right)^{\lambda+1} > \ldots > 0 \quad (27)$$

Therefore

$$2 > \frac{x+y}{z} > \frac{x^2+y^2}{z^2} > \ldots\ldots > \frac{x^{\lambda-1}+y^{\lambda-1}}{z^{\lambda-1}}$$

$$> 1 > \frac{x^{\lambda+1}+y^{\lambda+1}}{z^{\lambda+1}} > \ldots > 0 \ldots\ldots\ldots\ldots (28)$$

Hence

$$z^{\lambda-1} < x^{\lambda-1} + y^{\lambda-1} \ldots\ldots\ldots\ldots\ldots\ldots (29)$$

By (3_3) and the last equation

$$\xi^{\lambda} < \lambda z^{\lambda-1} < \lambda (x^{\lambda-1} + y^{\lambda-1}) \ldots\ldots\ldots\ldots (30)$$

Hence

$$\xi < \lambda (\alpha^{\lambda-1} + \beta^{\lambda-1} H^{\lambda-1}) \ldots\ldots\ldots\ldots\ldots (31)$$

where $H = n/\xi$. Thus

$$x < \lambda \alpha^{\lambda} + \lambda \alpha \beta^{\lambda-1} H^{\lambda-1}$$

$$\alpha^{\lambda} + d < \lambda \alpha^{\lambda} + \lambda \alpha \beta^{\lambda-1} H^{\lambda-1}$$

or

$$d < (\lambda - 1) \alpha^{\lambda} + \lambda \alpha \beta^{\lambda-1} H^{\lambda-1} \ldots\ldots\ldots\ldots (32)$$

Therefore

$$\gamma^{\lambda} - \alpha^{\lambda} - \beta^{\lambda} = 2d < 2(\lambda - 1)\alpha^{\lambda} + 2\lambda \alpha \beta^{\lambda-1} H^{\lambda-1}$$

Substituting for γ its value from (25), and developing it by the binomial theorem and reducing, we obtain

$$\lambda \beta^{\lambda-1} (\alpha + \delta) + \binom{\lambda}{2} \beta^{\lambda-2} (\alpha + \delta)^2 + \ldots\ldots < 2\lambda \alpha \beta^{\lambda-1} H^{\lambda-1}$$

$$\alpha + \delta < 2\alpha H^{\lambda-1} < 2\alpha$$

Therefore, as $\xi < \eta < \zeta$ by (3_3), (4_3), and (5_3), and the general assumptions $x < y < z$, $H < 1$,

$$\delta < \alpha \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (24)$$

We remember here that $\delta \equiv 0 \pmod{\lambda^2}$ from (19), and note that from (3_3)

$$\lambda x^{\lambda-1} < \lambda y^{\lambda-1} < \xi^{\lambda}$$

Therefore

$$\lambda \alpha^{\lambda} < x \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (25)$$

By (19) we see that δ always contains the factors 2 and λ^2. Hence by (24) and (25),

$$2^{\lambda} \lambda^{2\lambda+1} < \lambda \delta^{\lambda} < \lambda \alpha^{\lambda} < x < y < z \ldots\ldots\ldots\ldots\ldots (26)$$

[*Note.*—Mr. Spunar furnishes also a proof that δ always contains in addition the factor 3, which gives the still more stringent condition

$$6^{\lambda} \lambda^{2\lambda+1} < x < y < z$$

The proof of this is rather too long to be added here, but the fact is worthy of mention.—*P. R. Heyl.*]

In conclusion, I wish to give my best thanks to Dr. Paul R. Heyl for the great interest he has taken in preparing my paper for publication.

ETHNOLOGY.—*Some results of the study of American Indian music.*[1] FRANCES DENSMORE, Bureau of American Ethnology, Smithsonian Institution.

First of all, let me express my high appreciation of the honor of the invitation from M. Const. Maltezos, asking that I send him a note upon my study of Indian music, for transmission to the Academy of Athens. I am aware of M. Maltezos' deep and extended research in the music of ancient peoples, in both the old and new worlds. To that research he has brought the scholarly attainments of the physicist. My own work is done from the standpoint of a musician and an observer of human nature. Music is essentially a vital and human expression, especially in a primitive race like the American Indian. It has been said that "the North American Indians give us a fuller knowledge than any other existing race affords of the manner of

[1] Paper prepared by request and submitted at the sitting of the Academy of Athens, Greece, on March 22, 1928. The paper was translated by Professor Const. Maltezos and is being published by the Academy of Athens. Received May 4, 1928.

working of the primitive creative mind." The study of Indian music has, therefore, a relation to the general subject of primitive music in all races.

The Indians living in North America did not share the high development of those living in Central America and in certain parts of South America. There are no records in stone to tell us of their early art. The minds of the old men were the repositories of the wisdom and experience of the tribe, and it was the duty of the old men to transmit this orally to the next generation, each man instructing his oldest son. In development of memory the Indian excelled, but in logic and deductions from facts he did not excel. His culture and mental habit were not such as to produce a musical system comparable, for example, to that of the Hindu or the Chinese. The Indians are not a homogeneous people or nation but consist of many tribes which differ in language and important customs.

The following incident shows the manner of reasoning by a Sioux Indian who was highly respected among his people. He found a globular stone on top of a hill, similar to stones that were abundant in a river not far distant. On being asked how he explained the peculiar shape of the stone he said it had become globular by looking at the sun since "things that look at each other for a long time will come to have a resemblance." He carried this stone on his person and attributed the good health of himself and his family to its presence. In order to stimulate the supposedly magic power of the stone he sang a song, according to an Indian custom which will be described in this paper.

Agassiz, the great naturalist, said that "the function of science is to strive to interpret what actually exists." In the study of Indian music the facts are the vocal sounds produced by the Indians and recorded by means of the phonograph. The interpretation must concern itself with the life and customs of the Indian, and with his mental attitude. If we were to try to understand Indian songs without taking all these into consideration we would become involved in a maze of speculation. Although we admit the kinship of all humanity, the Indian belongs to a different race from our own. His habit of thought and his standard of beauty are not like ours, and even more different is his idea of the function of music in his daily life. Music, to the American Indian, was not primarily an art to be cultivated for pleasure. It was part of the means by which he exercised magic, and it lay, in part, in the field of religion.

In the oldest times of which we have any knowledge, the Indians believed that songs were received in dreams, and this continues to the present time among many old Indians. The "dream" of the Indian is a trance-like condition induced by abstinence from food and intense concentration of the mind. While in this condition the Indian imagines himself visited by a supernatural being which promises its aid in any difficulties that may befall him. The mysterious visitant usually sings a song which the dreamer learns and is told to sing when asking for the promised aid. On awaking, he remembers the song and it becomes his most valued possession. It can hardly be supposed that such songs are based upon an intelligent musical system. The profound students and thinkers among the Indians were concerned with means of obtaining supernatural help, not with calculations nor material facts. They were mystics and their old songs can not be separated from their mysticism.

A type of song which is probably as old as the "dream song" is that connected with folk-tales, many of which were of cosmic significance and intended for the instruction of the people. Melodies were introduced at intervals during a long story and were sung by the narrator. In the most primitive tribe under observation the melody was sung only once, after which the narrator resumed his story. It appears that the song broke the monotony of the narrative in an agreeable, rhythmic manner, and it usually represented the expression of some character in the story, either human or otherwise. For example, one story recorded by the writer was concerning a contest between a beaver and a plant called "fox-tail" as to which could cause rain to fall. Each sang his own magic song and the beaver produced the heavier downpour of rain. In explanation it was said that the beaver had a stronger supernatural helper than the flower. Can we expect to find in such material a consciously evolved musical system?

I began my study of Indian music in 1893, and have conducted that research for the Bureau of American Ethnology of the Smithsonian Institution since 1907. In the pursuit of this study I have visited many Indian reservations and recorded songs on the phonograph, transcribing and analysing about 1700 of such records. For transcriptions, I use the ordinary musical notation with only a few additional signs. If a tone is sung less than a quarter-tone higher than the indicated pitch I place a plus sign above the note, and if a tone is sung less than a quarter-tone below the indicated pitch a minus sign is placed above the note. A slight extending or diminishing of the length of the tone is indicated by a sign for a "hold," placed

vertically above the note, the opening of the curve being toward the left if the tone is shortened and toward the right if it is prolonged. By this simple and familiar notation it is possible to present a large amount of material in a form convenient for observation. A more elaborate graphic representation would make it necessary for students to master a new medium of expression and, if accurate and adequate, would require the services of a large staff of workers. The writer has no assistants and the opportunity to secure genuine Indian songs is rapidly passing away. These are among the circumstances which justify the use of ordinary musical notation in the transcription of Indian songs. It is not claimed that Indians sing all the tones of the diatonic scale with accuracy but it will be shown that the upper partials (overtones) of a fundamental tone constitute the framework of many Indian songs. These tones are usually sung with a degree of accuracy that would be considered acceptable in a cultured singer.

The Indian produces his tone in the back of his throat, holding the lips and teeth motionless and separating the tones by a peculiar action of the throat muscles. This tone is unfocused and frequently resembles the vocal sound produced by an animal. There is a multiplicity of by-tones which suggests that the song is progressing by minute intervals of pitch, and one of the first decisions that must be made by a student of Indian music is concerning the degree of importance to be attached to these small gradations of pitch.

It may safely be assumed that if exceedingly small intervals or gradations of pitch are consciously produced, the Indian must have an ability to discriminate such intervals when hearing them. In order to test the pitch-discrimination of Indians the writer took with her, to Indian reservations, a set of eleven standardized tuning forks, one of which gave the fundamental tone of the series (a′, 435 vibrations, international pitch) while the other forks produced tones respectively 1/2, 2, 3, 4, 5, 8, 12, 17, 23 and 30 vibrations above the fundamental. These forks were lent by Dr. C. E. Seashore, Dean of the Graduate College, University of Iowa, who kindly examined the tabulated report of the result of the test. He expressed the opinion that "the abilities here shown are about as good as one would find among the average American whites under similar conditions." The ear of the Indian is trained to hear sounds which we do not notice but this test does not indicate that he has a superior perception of differences in the pitch of tones. A practical experience with Indians has convinced me that, among uncultured people, small variations in the pitch of vocal sounds are not directed by an intelligence which would entitle

them to serious consideration. I do not think that a voluminous study of them would reveal any underlying system or laws. Lacking that definite purpose, the undertaking does not seem justified.

During the first year of my work with the recording phonograph I made an experiment which has an important bearing on this subject. Two phonographs were placed opposite each other in such a position that the ends of the recording horns were together. Selecting a typical record of an Indian song, I played it on one phonograph and recorded it on the other. A duplicate was made from this record and so on until I had the sixth duplication of the original record. This was much softer than the original but the tones were those of the diatonic scale, sung with reasonable accuracy. The duplications had eliminated the by-tones, leaving the kernel of tone which had been obscured by the Indian's manner of rendition. As so much depended upon the accuracy of my hearing I underwent a test by Dr. C. E. Seashore in his laboratory, the result being entirely satisfactory. This may be regarded as a standardizing of the human ear in order that it may safely be used as an instrument of measurement.

An Indian seldom strikes the drum or shakes his rattle precisely with the corresponding tone of his song. Through the courtesy of Dr. Dayton C. Miller, head of the department of physics, Case School of Applied Science, Cleveland, Ohio, this peculiarity of Indian music was given a graphic proof. The writer's phonograph was installed in Dr. Miller's laboratory, portions of two records were played by the phonograph and the sound recorded graphically by the phonodeik, an instrument of Dr. Miller's invention. In one of the songs thus studied, the portion photographed by the phonodeik was of about 23 seconds' duration, as reproduced by the phonograph, and made a film record about 38 feet in length. In a detailed report on this test Dr. Miller stated that "the first beat of the pair of drumbeats follows the beginning of an accented voice tone with great regularity. Of 25 such instances identified on the photograph the drumbeat follows the voice by 0.12 second in 12 cases and in no instance does the interval differ from this by more than 0.02 second." Other interesting results of this test are apart from our present consideration.

In order to determine whether Indian music resembles that of a European people, a comparison was made between the structure of Indian and Slovak songs, the latter being analysed by the same method employed in analysing Indian songs. The material used in this comparison consisted of 710 Indian songs from widely separated tribes and 10 typical Slovak songs selected for the purpose by Mr.

Ivan Daxner, secretary of the Slovanian League of America, whose co-operation is gratefully acknowledged. The Slovak songs included the Slovak national anthem, a song concerning Janosik, a very ancient melody entitled "In praise of song," a "dialogue on melody," several love songs, and folk-songs concerning the plowboy and the girl who watched the geese. On comparing the structural analyses it was found that the resemblances were fewer than the differences. The Indian and Slovak songs under analysis differed in trend and in the principal interval of progression; it also appeared that the Slovak songs had more directness in beginning and more simplicity of rhythm.

In order to obtain representative material it is necessary to exercise great care in the selection of Indian singers and interpreters. To this phase of the work one must bring a knowledge of Indians which can be gained only by experience. The Indians have their standards of good singing and recognise only one correct version of a song. This version must be obtained if the work is to be reliable. One of the requirements of a good singer among the Indians is that he shall repeat a song with absolute accuracy. To the credit of the Indians it may be said that repetitions after a period of weeks, months or (in one instance) after the expiration of two years have been found identical in tempo, pitch and note-values. Similarly, it is not unusual for as many as ten renditions of a song to be recorded on one phonograph cylinder without the slightest differences in the renditions. This accuracy appears in ceremonial songs and in records made by the best singers. There are other instances in which we find slight and unimportant variations in the renditions. The first is usually the best rendition in such cases but transcription is made from the clearest, wherever it occurs on the cylinder.

Mention has been made of a system of analysis devised and used by the writer. This consists at present of 18 tables of classification. Four additional tables were used in the analysis of 710 songs and the results were so similar in the tribes under analysis that these bases of classification were discontinued as being no longer necessary. The discontinued tables concerned the metronome tempo of voice and drum, a comparison of these tempi, and the pitch of the keynote of the song. The tables now in use are as follows:

1. Tonality (determined by the interval between the keynote and its third and sixth). 2. First note of song—its relation to keynote. 3. Last note of song—its relation to keynote. 4. Last note of song—its relation to compass of song. 5. Number of tones comprising compass of song. 6. Tone material. 7. Accidentals. 8. Structure (melodic or harmonic). 9. First progression—

downward and upward. 10. Total number of progressions—downward and upward. 11. Intervals in downward progression. 12. Intervals in upward progression. 13. Average number of semitones in an interval. 14. Part of measure on which song begins. 15. Rhythm (meter) of first measure. 16. Change of time (measure-lengths). 17. Rhythmic unit. 18. Rhythm of drum or rattle.

Each song is analysed according to these tables as soon as it is transcribed. These analyses are combined into a tribal group, and the tribal groups are, in turn, combined in a large total which shows the characteristics of all the songs under consideration. About 1700 songs have been transcribed and analysed but only 1073 have been combined in the large total. The data to be presented are based upon this group of 1073 songs, comprising songs of the Chippewa, Sioux, Ute, Mandan, Hidatsa, Papago, and Pawnee tribes. The songs analysed singly but not included in this total are those of the Yuma, Cocopa, Yaqui, Makah, Menominee, Winnebago, Tule Indians of Panama, and the Salish and Tsimshian songs recorded in British Columbia. A large number of Indian songs have been heard at tribal gatherings and not recorded phonographically. In all the tribes an effort has been made to obtain the oldest songs, recorded by the most reliable singers. Special attention has been given to songs connected with magic or with the treatment of the sick. A limited number of comparatively modern songs have been recorded for purposes of comparison.

The tables of analysis which chiefly interest us at present are Nos. 2, 3 and 6 from which the problem of scale in Indian music may be discussed. The term "keynote" is applied to the tone which, by the test of the ear, appears to be the fundamental tone in the series of tones comprising the song. This does not present a claim that the Indian regards it in this manner. The term is used for convenience, similarly to the ordinary notation in which the songs are transcribed. Having decided upon the tone to be designated as the keynote, the analysis of the song proceeds upon that basis. In a limited number of songs there is no apparent feeling for a keynote, such songs being pure melody without tonality. These occur chiefly in the songs of tribes which are not included in the present total. They are classified as irregular in tonality and await further study.

On examining Table 2 we find that 216 songs (20 percent) begin on the octave above the keynote, 150 songs (14 percent) begin on the twelfth, and 285 songs (27 percent) begin on the fifth above the keynote. The ratios of these intervals are respectively 2/1, 3/1, and

3/2. On examining Table 3 we find that the keynote is the final tone in 597 songs (56 per cent), and the fifth is the final tone in 342 songs (32 percent). The data in these two tables indicate a feeling for tones having simple vibration-ratios, such tones forming what may be termed the boundaries of the songs.

At this point it should be stated that the writer uses the term "scale" for convenience. It is applied only to the series of tones commonly known as the major and minor diatonic scales, appearing either in complete or incomplete form, and to the five five-toned scales which are designated by Helmholtz, appearing in complete form. The vibration ratios in the first-named are as follows:

Major diatonic scale

9/8, 10/9, 16/15, 9/8, 10/9, 9/8, 16/15,

Minor diatonic scale

9/8, 16/15, 9/8, 10/9, 16/15, 9/8, 10/9

The tone material of the songs is presented in Table 6 and, before considering the principal groups, mention should be made of a group of 71 songs (6 per cent) which contains a large variety of songs not otherwise classified. In this group are 42 songs which, in the first year of the writer's work, were transcribed in outline, no keynote being designated. There are 12 songs classified as irregular in tonality, and other series of tones appearing only twice in the 1073 songs. The largest subdivision of this group consists of 10 songs containing the first, second, fifth and sixth tones of the diatonic octave (vibration ratios 9/8, 4/3, 9/8, 6/5). This group may contain material of value but our present concern is with the larger groups.

The seven tones, or degrees, of the diatonic octave (major or minor) occur in 62 songs (5.3 per cent). Six tones occur in 204 songs (19 percent) and four tones in 319 songs (29 per cent). The number of tones preferred by the Indian is five, since 499 songs (46.4 percent) contain 5 tones or degrees of the diatonic octave.

Within the group of songs with five tones we find that 236 (47.1 percent) contain the series of tones designated by Helmholtz as the fourth five-toned scale and commonly known as the major pentatonic scale, having the following vibration-ratios:

9/8, 10/9, 6/5, 10/9, 6/5

Next in number are 106 songs (20 percent) which contain the series designated by Helmholtz as the second five-toned scale and commonly known as the minor pentatonic scale, having the following vibration-ratios:

6/5, 9/8, 10/9, 6/5, 10/9

A small group, 18 in number, contains the first five-toned scale according to Helmholtz which has the following vibration-ratios:

10/9, 6/5, 9/8, 10/9, 6/5

Only 2 songs are based on the fifth five-toned scale which has the following vibration-ratios:

6/5, 10/9, 6/5, 9/8, 10/9

The other songs containing 5 degrees of the diatonic scale are in many combinations of tones and are not regarded as on any scale. In 107 of these five-toned songs (not on the pentatonic scales designated by Helmholtz) the seventh and one other degree of the octave are absent. The total of 1073 songs does not show this group in detail but in a previous total of 820 songs we may examine the number of songs in which two degrees of the diatonic octave are absent. In this we find the largest group to be 29 songs which lack the seventh and sixth degrees, this series having the vibration-ratios 9/8, 10/9, 16/15, 9/8, 4/3. Next to the largest group comprises 26 songs which lack the seventh and second degrees, this series having the vibration-ratios 5/4, 16/15, 9/8, 10/9, 6/5. The general proportions would be the same if this detailed observation were extended to the analysis of 1073 songs. In the five-toned songs we find that 19 omit the sixth and one other tone, while in 10 songs the fourth and one other tone do not occur. It is, therefore, apparent that the seventh degree of the octave is omitted in a larger number of these Indian songs than any other degree of the octave.

Table 7 is devoted to a consideration of accidentals, or tones chromatically altered. In 923 songs (86 per cent) there are no accidentals. In 25 songs (minor in tonality) the seventh is raised a semitone, and in 22 songs the fourth is similarly raised. These are the largest groups containing only one accidental. Songs containing two accidentals number 89 and are of so many sorts that a detailed consideration of the group is not practical at this time.

From the foregoing it appears that tones with simple vibration-ratios are preferred by the Indians under observation as the boundary and the tone material of their songs, also that the seventh is the scale-degree most frequently omitted, and the seventh and fourth are the scale-degrees most often altered chromatically in the songs.

It is not enough for our purpose, however, to consider the tones used by the Indians in their songs. Equally important is the sequence in which these tones are used. For example, Indian songs containing

the tones of the "major pentatonic scale" (fourth five-toned scale) do not resemble the Gaelic songs based on that scale, and the tone material frequently is not discerned until the song is analysed. The sequence of tones is shown in Tables 11 and 12 in which the number and sort of intervals are considered. These tables show that the interval occurring most frequently is the whole tone, or major second (9/8). The entire number of intervals in the 1073 songs under analysis is 28,956, and the whole tone progressions comprise 11,741 (40 percent) of that number. Next in frequency is the minor third (6/5) which comprises 8,029 (29 percent). The interval of a fourth (4/3) constitutes 3,717 (13 percent), and the semitone, or minor second (16/15), constitutes 1,117 (4 per cent). Attention is directed to the relatively small number of semitone progressions, which does not encourage the hypothesis that the Indian possesses a musical system consisting of small intervals or gradations of pitch. The intonation on the semitone is more variable than on any other interval. The tone transcribed as a minor third is more frequently a non-major third than a correct minor third. It is interesting to note that the major third comprises only 2,932 (10 per cent) of the total number, as that interval is part of the harmonic series already shown to be prominent in the framework of the melodies.

The interval of a fourth has been found to occur most frequently in songs concerning birds, animals and motion. Indeed, it may be said to characterise such songs. For example, it is a prominent interval in songs with the following titles which are derived from the words of the songs: "The ravens are singing," "The big bear, to his lodge I often go," "I am raising my pipe," and "A bubbling spring comes from the hard ground." The fourth may be called a progressive interval, implying action that is to be completed.

The foregoing general observation on a large number of intervals is, in the writer's opinion, more important than an intensive study of a limited number of intervals. Indian music is mental in its origin and the Indian evidently "thinks" most clearly the intervals which have simple vibration-ratios. His deviations involve a personal element. To follow such deviations is apart from our present purpose.

We will now proceed to a peculiarity of Indian songs which can hardly be tabulated but which is observed when the songs are studied. This may be termed an "interval formation" and has no counterpart in the music of civilization. It occurs most frequently in songs that lack a keynote and are classified as irregular in tonality. Perhaps, at some future time, the basis of classification may be changed and the

songs divided into those which have a keynote and those which are formed by successive intervals. In many of the latter songs a few measures are within a small compass, the next few measures are within a different compass, and so on to the end of the song with no binding intervals that would unite these small groups in relation to a common keynote. For example, a song for the treatment of the sick, recorded by Pigeon-hawk, begins with a few measures on the descending fourth A to E, these are followed by a few measures on the descending fourth G to D, and the song closes with measures on the tones D to A. Each phrase is complete in itself, like a separate melody. In the songs of the Northwest Coast, not included in the present collective analysis, this peculiarity appears as a whole-tone formation, many songs having a compass of three or four tones with phrases based on whole tones, repeated within that compass. In such songs the feeling for a keynote is less prominent than the stepping from one tone to another which is adjacent or desirable for some other reason. This indicates the complexity of Indian music and the danger which lies in the making of generalizations.

It is difficult to trace the history of an Indian song more than 150 years and, with the utmost care in selection, we cannot be certain that every recorded melody is of purely Indian derivation. The Indians are imitative and when visiting other localities they are particularly eager to hear and learn new songs. In this manner there may have been alien influences in their music which they do not recognise. The Yaqui Indians, living on the Mexican border, said they had two sorts of songs, one of which was their own and the other was "like Mexican music." Examples of both were recorded, the former being ceremonial songs and the latter being modern songs accompanied by the guitar. One song was recorded on Cape Flattery on the northwestern boundary of the United States, which closely resembles the Cocopa dancing songs recorded in southern Arizona by members of a tribe which lives partly in the United States and partly in Mexico. It is said that long ago the Spaniards visited Cape Flattery and this coincidence is interesting. Other songs recorded on Cape Flattery resemble chants, yet are not like the chanting in the Roman Catholic Church and we are reminded that the Russian Church is widely established in the far north. Possibly this may have influenced the Indians from Cape Flattery on some of their journeyings. Songs recorded by Indians living on the west coast of British Columbia are characterised by a fluency and easy tunefulness that bears no resemblance to typical Indian songs but is like the easy,

pleasing melodies used in missions of the Roman Catholic Church. The writer was not surprised to learn that the Roman Church has missionaries in that region and that certain of the recorded songs were used in Christmas festivities. The presence of such songs demonstrates the necessity for collecting a large amount of material on which to base conclusions.

The most conservative songs of the Indians are those connected with old ceremonies, the songs hereditary in families, the songs received in fasting dreams, and the songs used in the treatment of the sick. A very large majority of the songs collected and analysed by the writer belong to these classes, but other songs are also collected in order to secure a complete representation of a region and to afford material for comparison. If modern songs should be found to predominate in a region the writer would go elsewhere to collect songs. The desirability of a field for research is estimated by the number of medicine men still living and by the extent to which the native religious customs are observed. Distance from towns is also a factor. For example, the village on Cape Flattery where work has been conducted for two seasons is reached only by water and there is only one steamer a week, carrying freight and a few passengers. In winter this boat is often unable to land because of the high waves. The isolation of this village adds value to its musical material. Other songs have been recorded in a mountain village about 130 miles from a railroad. The Indians came a considerable distance to this village in order that their songs might be recorded phonographically. On the southern desert the writer has travelled more than 80 miles from a town or a telephone to record songs from the Indians, once spending Christmas in such a locality and attending a remarkable dance that was held only on Christmas night. This dance was given on the desert sand and was a wonderful sight in the moonlight. Many similar incidents could be related but do not concern our present purpose.

Mention may here be made of the use of a rest in Indian songs. A pause for the taking of breath is noted only in records made by younger Indians. It occurs rarely and can be distinguished from a musical rest as it does not occur uniformly in all renditions of the song. A rest occurred in only 10 Chippewa songs (less than one-half of one per cent) but was found in 13 (more than 11 per cent) of the Ute songs, being given with care and distinctness. It is used to a much larger extent in the songs of southern Arizona and in those recorded on Cape Flattery. If we were to form our opinion of Indian songs

according to those of the Chippewa we might imagine a resemblance to the songs of the Hindu concerning which it is said that "Rests are seldom written (except in order to break up the meter intentionally in a dramatic way) in any of their songs. . . . . They appear to take breath when they want to take it, not at the end of words."[2] Further experience shows that such a conclusion would be erroneous, as rests are effectively used in the songs of many Indian tribes.

The suggestion that Indian melodies are based on the tones produced by an instrument is untenable, as the only instruments used by the Indians that produce tones of different pitch are the flute and whistle. The first was formerly made according to the physical measurements of the man who was to play it, and the second produces two tones by a peculiar manner of blowing it. Neither instrument produces tones of sufficient accuracy to form the basis for songs.

The rhythm of Indian songs is characterised by accents unevenly spaced, transcribed as measures of different lengths. The tempo of the voice and the accompanying drum or rattle are frequently different, yet each is steadily maintained. When the tempo is the same, they frequently do not synchronize, as proven by the test conducted by Dr. Dayton C. Miller and described in this paper.

SUMMARY: The music of the American Indians is formed and preserved mentally, not visually. The collective analysis of 1073 songs shows a perception of tones with simple ratios of vibration. These tones, however, are frequently used in what may be termed an "interval-formation" of melody which does not suggest a keynote and has no counterpart in our musical usage. The analysed songs of the North American Indians do not suggest a resemblance to the songs of Asiatic or European countries. Interesting resemblances to less distant music are occasionally noted but are considered less important than the larger data obtained through collective analysis. A group of songs now designated as irregular in tonality is reserved for further study.

The rhythm of Indian songs shows more striking peculiarities than the melodic progressions. In this connection we repeat the statement that Indian music was originally associated with the working of magic and the treatment of the sick and that its use in the old manner persists, in many localities, until the present day.

The music of the American Indian is not an art, in our use of the term, but is primarily a means by which the Indian believes that he

[2] FOX STRANGWAYS, *Music of Hindustan*, pp. 192–193.

puts himself in communication with the mysterious forces of the earth, sea, and air. These are beneficent forces, though he regards them with awe and reverence, and he looks to their aid for safety in his daily life and for success in his undertakings. The study of Indian music cannot be separated from a study of the Indian himself, his traditions and his highest beliefs.

ZOOLOGY.—*A new francolin from Abyssinia.*[1] HERBERT FRIEDMANN, Amherst College. (Communicated by A. WETMORE.)

Among the birds collected by the late Dr. Edgar A. Mearns while on the Childs Frick Expedition to Abyssinia and East Africa, 1911–12, are two examples of a francolin unlike any known form. They are nearest to *Francolinus africanus psilolaemus*, but are much darker and more abundantly and heavily marked with both rufous brown and with black on the breast, abdomen, and flanks. This new form I propose to name in honor of the leader of the expedition

Francolinus africanus fricki subsp. nov.

Type: U. S. N. M. no. 243201, adult ♂, collected in the Arussi Plateau (altitude 10,500 ft.), Abyssinia, 18 February 1912, by Edgar A. Mearns.

Subsp. Characters: Similar to *psilolaemus* G. R. Gray, but much darker above, the feathers largely blackish, the underparts as in *psilolaemus* but the breast, abdomen, and flanks heavily marked with arrow-shaped black, subterminal marks as well as with broad spots of deep rufous brown, while in *psilolaemus* the black marks are very few and narrow, only the rufous brown ones being abundant; also the size is larger (wing ♂ and ♀ 174 mm. as against 164–167 mm. in *psilolaemus*).

Measurements of type: wing 174 mm.; tail 81 mm.; culmen from base 27 mm.

Measurements of ♀ (*topotype*)*:* wing 174 mm.; tail 83 mm.; culmen from base 26.5 mm.

Range: known only from the type and topotype, both of which come from the heath zone at 10,500 feet, Arussi Plateau, Abyssinia. According to Dr. Mearns' field notes, it is abundant up to 11,000 feet.

Remarks: I have been able to compare these two birds with but a single example of *psilolaemus*, an adult female from Antoto, Abyssinia, and with the colored plate in the Catalogue of Birds in the British Museum,[2] with which the Antoto specimen agrees very closely.

The female of *fricki* is similar to the male but is very slightly darker above, has more of the black markings and less of the reddish brown ones below.

It seems that *fricki* is the representative of *psilolaemus* in the very high districts of the Arussi country.

[1] Received July 9, 1928.

[2] Vol. 22, *pl. 3.* 1893.